One-loop effective action for Einstein gravity in special background gauge

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Abstract

The one-loop effective action for Einstein gravity in a special one-parameter background gauge is calculated up to first order in a gauge parameter. It is shown that the effective action does not depend upon the gauge parameter on shell.

1. Introduction

The field models are recently formulated, as a rule, in the form of gauge theories (electrodynamics, chromodynamics, (super)gravity, (super)string, etc.). It is well known, since quantization of such theories involves introducing the gauge, that the Green's functions, possessing the whole information about quantum properties of the theory, depend on choice of the gauge (see for example [1]). On the other hand, physical values (in particular, the S matrix) must not depend on choice of the gauge. This fact implies that gauge dependence in gauge theories is a special issue [2,3]. The most

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detailed study of the problem in question for the case of general gauge theories with arbitrary gauges in the framework of standard Lagrangian BRST quantization [4] is given in ref.[5]. Meanwhile, the corresponding generalization to the case of the extended BRST quantization method [6] is presented in ref.[7].

The study of gauge dependence for concrete field models is currently popular [8-14], but the results presented are not always correct (see for example refs.[15-17]). Our attention to the problem in question is due to ref.[17], which presents calculation of the one-loop effective action within a special class of one-parameter background gauges. In ref.[17] it is stated that the one-loop effective action depends manifestly on choice of the gauge on shell. This result is in contradiction with general assertions of papers [3,5].

In this connection, the present paper deals with calculation of the one-loop effective action for Einstein gravity within the class of gauges suggested in ref.[17]. Discrepancy with the result given in ref.[17] is found along with the source bringing it about. The effective action is shown to depend on choice of the gauge in a manner, being in accordance with the statements of refs.[3,5].

In this paper we use the condensed notations suggested by De Witt [18]. The Grassmann parity of a quantity G is denoted $\varepsilon(G)$. Derivatives with respect to fields $A^\imath$ are always understood as right and those with respect to sources $J_\imath$ as left. For derivatives with respect to $A^\imath$ we use the special notation $F,_\imath(A) \equiv \delta F(A)/\delta A^\imath$. In what follows, we use the terminology, now becoming generally accepted as regards gauge field theories.

2. Gauge dependence of effective action in Einstein gravity

Let us consider the Einstein theory of gravity described by the classical action

$$\mathcal{S}(\bar{g}_{\mu\nu}) = -\frac{1}{k}\int d^4x\sqrt{-\bar{g}}\bar{R} \tag{1}$$

$(g_{\mu\nu} = diag(-,+,+,+), R^{\mu}_{\nu\alpha\beta} = \partial_\alpha \Gamma^{\mu}_{\nu\beta} - ..., R_{\alpha\beta} = R^{\mu}_{\alpha\mu\beta}, R = R^{\alpha\beta} g_{\alpha\beta})$, k is the gravitational constant.

The action (1) is invariant under the general coordinate transformations:

$$\delta \bar{g}_{\mu\nu} = \Big(\bar{g}_{\mu\sigma} \nabla_\nu(\bar{g}) + \bar{g}_{\nu\sigma} \nabla_\mu(\bar{g}) \Big) \eta^\sigma \equiv \vec{H}_{\mu\nu\sigma}(\bar{g}) \eta^\sigma \tag{2}$$

(with an arbitrary vector η^σ)

$$\mathcal{S},_\imath (\bar{g}) \mathcal{R}^\imath_\alpha(\bar{g}) \equiv \frac{\delta \mathcal{S}(\bar{g})}{\delta \bar{g}_{\mu\nu}} \vec{H}_{\mu\nu\sigma}(\bar{g}) \equiv 0 \ , \tag{3}$$

here we use the condenced notations $\mathcal{S},_\imath (\bar{g})$ and $\mathcal{R}^\imath_\alpha(\bar{g})$ for the classical equations of motion and the generators of the gauge transformations respectively. In (3) we have also introduced, for the sake of convenience, the following condensed notations:

$$\imath = (\mu, \nu, x), \quad \alpha = (\sigma, y), \tag{4}$$

where μ, ν, σ, are the Lorentz indices, x, y, z are the space-time coordinates of the Riemann manifold.

The algebra of the local generators $\vec{H}_{\mu\nu\sigma}(\bar{g})$ is closed, with the structural coeffitients not dependnig upon the fields $\bar{g}_{\mu\nu}$ and having the form

$$f^\lambda_{\rho\sigma}(x, y, z) = \delta^\lambda_\sigma \delta(x-y) \partial_\rho \delta(x-z) - \delta^\lambda_\rho \delta(x-z) \partial_\sigma \delta(x-y) \ . \tag{5}$$

Since the Faddeev-Popov rules could be applied to the theory in question, the nonrenormalizable generating functionals of the Green's functions $Z(J)$ and the vertex functions $\Gamma(\bar{g})$ are given, with allowance made for the condensed notations (4), in the form

$$\begin{aligned} Z(J) &= \int D\bar{g} \exp\Big\{ \frac{\imath}{\hbar} \Big(S_\psi(\bar{g}) + J_\imath \bar{g}^\imath \Big) \Big\} \ , \\ \Gamma(\bar{g}) &= \frac{\hbar}{\imath} \ln Z(J) - J_\imath \bar{g}^\imath \ , \quad \bar{g}^\imath = \frac{\hbar}{\imath} \frac{\delta \ln Z(J)}{\delta J_\imath} \ , \end{aligned} \tag{6}$$

where $S_\psi(\bar{g})$ is the quantum action constructed by the rules:

$$\begin{aligned} & S_\psi(\bar{g}) = \mathcal{S}(\bar{g}) - \frac{1}{2} \chi_\alpha(\bar{g}) g^{\alpha\beta} \chi_\beta(\bar{g}) - \imath\hbar \mathrm{Tr} \ln M(\bar{g}) \ , \\ & M_{\alpha\beta}(\bar{g}) = \chi_{\alpha,\imath}(\bar{g}) \mathcal{R}^\imath_\beta(\bar{g}) \ . \end{aligned} \tag{7}$$

Here $\chi_\alpha(\bar g)$ is a gauge function supposed to be linear in the fields $\bar g_{\mu\nu}$, while $\hbar$ is the Plank constant.

In the framework of the background-field method $\bar g_{\mu\nu}$ can be represented in the form:

$$\bar g_{\mu\nu} = g_{\mu\nu} + \sqrt{k} h_{\mu\nu} \ , \tag{8}$$

where $g_{\mu\nu}$ is the background part of the complete field $\bar g_{\mu\nu}$, satisfying the classical equations of motion, and $h_{\mu\nu}$ is the quantum field.

By virtue of decomposition (8), the gauge transformations can be written in the form

$$\begin{aligned} \delta g_{\mu\nu} &= 0 \\ \sqrt{k}\delta h_{\mu\nu} &= \Big(\bar g_{\mu\sigma}\nabla_\nu(\bar g) + \bar g_{\nu\sigma}\nabla_\mu(\bar g)\Big)\eta^\sigma \equiv \vec{H}_{\mu\nu\sigma}(\bar g)\eta^\sigma \end{aligned} \tag{9}$$

Now choose for the action (1) the gauge condition in the form of a special one-parameter background gauge [17]

$$\begin{aligned} \chi^\rho(g,h,\zeta) = \Big\{ \frac{1}{2}\Big[g^{\rho\tau}\nabla^\sigma + g^{\rho\sigma}\nabla^\tau - \frac{1}{2} g^{\sigma\tau}\nabla^\rho \Big] \\ + \zeta \cdot k \cdot R^{\tau\omega\sigma\rho}(g)\nabla_\omega \Big\} h_{\tau\sigma} \ , \end{aligned} \tag{10}$$

where ζ is the gauge parameter.

Let us now introduce the operator $L^{\rho\sigma,\mu\nu}(\zeta)$ necessary for the calculation of the one-loop counterterms to the effective action. It is defined by the part of the complete quantum action S_ψ quadratic in the fields $h_{\tau\sigma}$ and minimal when $\zeta = 0$:

$$\begin{aligned} L^{\rho\sigma,\mu\nu}(0) &= \frac{\delta}{\delta h_{\rho\sigma}} \frac{\delta_l}{\delta h_{\mu\nu}} \Big\{ \mathcal{S}(\bar g) \\ &- \frac{1}{2}\int d^4x \sqrt{-g}\chi^\rho(g,h,0) g_{\rho\sigma}\chi^\sigma(g,h,0) \Big\}_{|h=0} \\ &= \sqrt{-g} C^{\rho\sigma,\lambda\delta}\Big\{ \Box \delta^{\mu\nu}_{\lambda\delta} + P^{\mu\nu}_{\lambda\delta} \Big\} \ , \end{aligned} \tag{11}$$

where

$$\delta^{\mu\nu}_{\lambda\delta} = \delta^\mu_{(\lambda}\delta^\nu_{\delta)} \ ,$$

$$C^{\rho\sigma,\lambda\delta} = \frac{1}{4}\Big(g^{\rho\lambda}g^{\sigma\delta} + g^{\rho\delta}g^{\sigma\lambda} - g^{\rho\sigma}g^{\lambda\delta}\Big) \ ,$$

$$P^{\mu\nu}_{\lambda\delta} = 2R^{(\mu}_{\lambda\ \delta}{}^{\nu)} + 2\delta^{(\mu}_{(\lambda}R^{\nu)}_{\delta)} - g^{\mu\nu}R_{\lambda\delta} - g_{\lambda\delta}R^{\mu\nu} - R\delta^{\mu\nu}_{\lambda\delta} + \\ + \frac{1}{2}g_{\lambda\delta}g^{\mu\nu}R \ , \qquad (12)$$

the symbol $\frac{\delta_l}{\delta h_{\mu\nu}}$ denotes the left derivative with respect to the field $h_{\mu\nu}$, whereas the indices in brackets imply symmetrization with a factor $\frac{1}{2}$. The Green's functions of the gauge and ghost fields $G_{\mu\nu,\lambda\delta}(\zeta)$ and $Q^{\sigma}_{\rho}(\zeta)$ are defined when $\zeta = 0$ by the relations:

$$L^{\rho\tau,\mu\nu}(0)G_{\mu\nu,\lambda\sigma}(0) = -\delta^{\rho\tau}_{\lambda\sigma} \ ,$$

$$\Big(\vec{H}_{\mu\nu\rho}(g,h)\frac{\delta\chi^{\sigma}(g,h,0)}{\delta h_{\mu\nu}}\Big) \cdot Q^{\tau}_{\sigma}(0) = \delta^{\tau}_{\rho} \ . \qquad (13)$$

Here the operator $L^{\rho\tau,\mu\nu}$ is written without $\sqrt{-g}$ and the Faddeev-Popov matrix $M_{\alpha\beta}$ in (7) has the form

$$M^{\sigma}_{\rho}(g,h,\zeta) = \vec{H}_{\mu\nu\rho}(g,h)\frac{\delta\chi^{\sigma}(g,h,\zeta)}{\delta h_{\mu\nu}} \ . \qquad (14)$$

One-loop effective action of the theory is given by:

$$\imath\Gamma_1(\zeta) = -\frac{1}{2}\mathrm{Tr}\ \ln L^{\rho\tau,\mu\nu}(\zeta) + \mathrm{Tr}\ \ln M^{\sigma}_{\rho}(\zeta) \ . \qquad (15)$$

Differential consequence of the identities (3) and the relations (13) lead to the one-loop Ward identity for the Green's functions (the condensed notations are used)[19]:

$$\frac{\delta\chi_{\alpha}(\zeta)}{\delta h^n}G^{nm}(\zeta) = Q^{\beta}_{\alpha}(\zeta)\mathcal{R}^m_{\beta} - \mathcal{S}_{,\imath}\frac{\delta\mathcal{R}^{\imath}_{\beta}}{\delta h^n}G^{nm}(\zeta)Q^{\beta}_{\alpha}(\zeta) \ . \qquad (16)$$

From the identities (16) there follows the representation for the one-loop effective action with an accuracy up to the first order in the gauge parameter:

$$\imath\Gamma_1(\zeta) = \imath\Gamma_1(0) + \zeta\cdot\mathcal{S}_{,\imath}\frac{\delta\mathcal{R}^{\imath}_{\alpha}}{\delta h^n}G^{nm}(0)Q^{\alpha}_{\beta}(0)\Big(\frac{d}{d\zeta}\frac{\delta\chi^{\beta}(\zeta)}{\delta h^m}\Big) \\ + O(\zeta^2) \ . \qquad (17)$$

Note that calculation of the counterterms for divergent structures in (17) involves gauge invariant regularization for the Einstein gravity (namely, dimensional). Absence of anomalies for the general coordinates invariance is also taken into account.

For calculation of a divergences in (17) we applied the Barvinsky-Vilkovisky diagrammatic technique in the dimensional regularization scheme [19] (assuming $\delta(0) = 0$) of the Schwinger proper-time integration method.

All diagrams the representation (17) for $\Gamma_1(\zeta)$ contains are finite with the background dimensionalities $O(\frac{1}{l^n}), n > 4$. There are the following background dimensionalities of values in the theory $[\mathcal{S}_{,\imath}] = [R^{\mu}_{\ \nu\alpha\beta}] = [R_{\alpha\beta}] = [R] = [\frac{1}{k}] = O(\frac{1}{l^2})$. Calculation of divergences in (17) gives a quadratically divergent diagram and two logarithmically divergent diagrams:

$$
\begin{aligned}
K_{1|div} &= -\zeta \cdot k \int \Gamma^{(\rho}_{\mu\alpha}(\nabla)\mathcal{L}^{\mu\sigma)}\nabla_\tau R^{\lambda\tau\delta\alpha} C^{-1}_{\rho\sigma,\lambda\delta} \\
&\quad \frac{\hat{1}}{\Box^2}\delta(z-y)dz_{|y=z,div}, \\
I_{1|div} &= -\zeta \cdot k \int \mathcal{L}^{\mu(\sigma} R^{\lambda\tau\delta\gamma} R^{\alpha}_{\gamma} \nabla_\tau \Gamma^{\rho)}_{\mu\alpha}(\nabla) C^{-1}_{\rho\sigma,\lambda\delta} \\
&\quad \frac{\hat{1}}{\Box^3}\delta(z-y)dz_{|y=z,div}, \\
I_{2|div} &= -\zeta \cdot k \int \mathcal{L}^{\mu(\sigma} R^{\lambda\tau\delta\alpha} P_{\rho\sigma,\lambda\delta} \nabla_\tau \Gamma^{\rho)}_{\mu\alpha}(\nabla) \\
&\quad \frac{\hat{1}}{\Box^3}\delta(z-y)dz_{|y=z,div},
\end{aligned}
\tag{18}
$$

where $\mathcal{L}^{\mu\sigma} = -\Big(R^{\mu\sigma} - \frac{1}{2}g^{\mu\sigma}R\Big)$ is the classical extremal,

$$
\begin{aligned}
\Gamma^{\rho}_{\mu\alpha}(\nabla) &= \delta^{\rho}_{\mu}\nabla_\alpha - 2\delta^{\rho}_{\alpha}\nabla_\mu, \\
C^{-1}_{\rho\sigma,\lambda\delta} &= \Big(g_{\rho\lambda}g_{\sigma\delta} + g_{\rho\delta}g_{\sigma\lambda} - g_{\rho\sigma}g_{\lambda\delta}\Big),
\end{aligned}
$$

and the operator $P_{\rho\sigma,\lambda\delta}$ is defined in (12).

The relations (18) are reduced to a table of the universal functional traces [19]. We only use the following ones:

$$
\nabla_\mu\nabla_\nu \frac{\hat{1}}{\Box^2}\delta(x-y)_{|y=x,div} = \frac{\imath \ln L^2}{16\pi^2}\sqrt{-g}\Big\{\frac{1}{6}\Big(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\Big)\hat{1}
$$

$$+\frac{1}{2}\hat{R}_{\mu\nu}\Big\},$$

$$\nabla_{\mu_1}...\nabla_{\mu_{2n-3}}\frac{\hat{1}}{\Box^n}\delta(x-y)_{|y=x,div}=0, \qquad n\geq 2,$$

$$\nabla_\mu\nabla_\nu\frac{\hat{1}}{\Box^3}\delta(x-y)_{|y=x,div}=\frac{\imath \ln L^2}{64\pi^2}\sqrt{-g}g_{\mu\nu}\hat{1}. \tag{19}$$

Here $\hat{R}_{\mu\nu}B=[\nabla_\mu,\nabla_\nu]B$, L^2 is the parameter of effective cutoff.

In view of (17), the resultant form for the divergent part of one-loop effective action

$$\begin{aligned}
&\Gamma_{1,div}(\zeta)=\Gamma_{1,div}(0)+K_{1|div}+I_{1|div}+I_{2|div}=\\
&=\frac{\imath \ln L^2}{16\pi^2}\int d^4x\sqrt{-g}\Big\{\frac{53}{15}\Big(R^2_{\mu\nu\alpha\beta}-4R^2_{\mu\nu}+R^2\Big)+\frac{21}{10}R^2_{\mu\nu}+\\
&+\frac{1}{20}R^2+\zeta\cdot k\Big[-6R_\tau^{\ \sigma}R^{\lambda\tau\delta\alpha}R_{(\lambda\alpha\delta)\sigma}+\frac{29}{2}R^{\rho\sigma}\Big(R_{\rho\ \sigma}^{\ \tau\ \ \alpha}R_{\alpha\tau}\\
&+R^\tau_{\ \sigma}R_{\rho\tau}\Big)+3RR^{\lambda\sigma\delta\alpha}R_{(\lambda\sigma\delta)\alpha}-\frac{67}{4}RR^2_{\mu\nu}+\frac{15}{8}R^3\Big]\Big\}\\
&\qquad\qquad +O(\zeta^2)
\end{aligned}\tag{20}$$

(calculation of $\Gamma_{1,div}(0)$ was given, for instance, in [19]) depends upon the gauge parameter ζ off-shell only. This result, being the consequence of general theorem for dependence of effective action upon the gauge in gauge theories, is not unexpected.

3. Effective action in general gauge theories

Let us consider the gauge theory of fields $A^\imath(\varepsilon(A^\imath)\equiv\varepsilon_\imath)$ with the classical action $\mathcal{S}(A)$ invariant under the gauge transformations of the form: $\delta A^\imath=\mathcal{R}^\imath_\alpha(A)\xi^\alpha$ (ξ^α are arbitrary functions, $\varepsilon(\xi^\alpha)\equiv\varepsilon_\alpha$), where $\mathcal{R}^\imath_\alpha(A)$ are generators of gauge transformations assumed to obey the usual requirements of irreducibility and completeness.

As mentioned above, the most detailed and complete investigation of gauge dependence in general gauge theories in the Lagrangian formulation of standard BRST quantization [4] was given in paper [5]. From the results [5] obtained in the framework of the

standard assumptions (a gauge invariant regularization, absence of anomalies) we only borrow those to relate immediately to the problem in question, i.e. the one of gauge dependence of Green's functions. In [5] it is proved that both nonrenormalizable and renormalizable generating functionals of vertex functions do not depend upon the gauge on their extremals (in particular, the renormalizable physical S matrix does not depend upon the gauge). This statement is valid for arbitrary gauge theories (when the algebra of gauge transformations is both closed and open) in arbitrary gauges.

In what follows it suffices for the purposes of this paper to confine ourselves to consideration of the Yang-Mills type theories. In terms of the generators of gauge transformations $\mathcal{R}^{\imath}_{\alpha}(A)$ these theories are formulated as gauge ones, for which the Lie brackets of generators $\mathcal{R}^{\imath}_{\alpha}(A)$ (commutator) have the form:

$$\mathcal{R}^{\imath}_{\alpha,\jmath}(A)\mathcal{R}^{\jmath}_{\beta}(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta}\mathcal{R}^{\imath}_{\beta,\jmath}(A)\mathcal{R}^{\jmath}_{\alpha}(A) = -\mathcal{R}^{\imath}_{\gamma}(A)F^{\gamma}_{\alpha\beta}, \tag{21}$$

with the structural coeffitients $F^{\gamma}_{\alpha\beta}$ not depending upon the fields $A^{\imath}$. The generators themselves form a complete and linearly independent set. They are also linear with respect to the fields $A^{\imath}$. For such theories one can specify the following result of ref.[5] obtained for the first time in [3].

The quantum action of the theory is constructed by the Faddeev-Popov rules:

$$\begin{aligned} &S_{\psi}(A) = \mathcal{S}(A) - \frac{1}{2}\chi_{\alpha}(A)\chi^{\alpha}(A) - \imath\hbar\mathrm{Tr}\,\ln M(A), \\ &M_{\alpha\beta}(A) = \chi_{\alpha,\imath}(A)\mathcal{R}^{\imath}_{\beta}(A). \end{aligned} \tag{22}$$

Here $\chi_{\alpha}(A)$ is the gauge function. In what follows we shall suppose it to be linear with respect to the fields $A^{\imath}$ (linear gauges). The generating functionals of Green's functions $Z(J)$ and vertex functions $\Gamma(A)$ are constructed by the following rules:

$$\begin{aligned} Z(J) &= \int DA \exp\left\{\frac{\imath}{\hbar}\Big(S_{\psi}(A) + J_{\imath}A^{\imath}\Big)\right\}, \\ \Gamma(A) &= \frac{\hbar}{\imath}\ln Z(J) - J_{\imath}A^{\imath}, \quad A^{\imath} = \frac{\hbar}{\imath}\frac{\delta \ln Z(J)}{\delta J_{\imath}}, \end{aligned} \tag{23}$$

where $J_\imath$ are the sources to the fields $A^\imath$.

The study of gauge dependence of $Z(J)$ and $\Gamma(A)$ is based on the fact that any variation of gauge conditions in (23) leads to a change of both the summand fixing the gauge $\frac{1}{2}\chi_\alpha\chi^\alpha$ and the summand containing the Faddeev-Popov matrix $M_{\alpha\beta}$. Both variations can be compensated by the corresponding gauge transformation of the fields $A^\imath$, which can be considered as the change of variables in the functional integral (23) with the Berezenian equal to 1. Thus, one can obtain an equation, determining gauge dependence for $\Gamma(A)$. Analysis of this equation implies [3] for the completely renormalizable quantum action

$$S_{\psi_R}(A) = S_\psi(A) - \textstyle\sum_{n=1}^{\infty}\Gamma_{n,div},$$

where $\Gamma_{n,div}$ is the divergent part of n-loop approximation for $\Gamma(A)$, which can be written in the form:

$$S_{\psi_R}(\{\vartheta\}, A) = \hat{S}_{\psi_R}(A'), \quad A' = A'(\{\vartheta\}, A) \tag{24}$$

($\{\vartheta\}$ is the set of all gauge parameters in $\chi_\alpha(A)$). Here all dependence upon the gauge is contained in the variables of gauge invariant functional $\hat{S}_{\psi_R}(A')$. In turn, for renormalizable generating functional of vertex functions Γ_R one can establish (in the class of linear gauges!) the following representation:

$$\Gamma_R(\{\vartheta\}, A) = \hat{\Gamma}_R(A') - \frac{1}{2}\chi_\alpha\chi^\alpha, \quad A' = A'(\{\vartheta\}, A), \tag{25}$$

where (excepting the gauge condition) all dependence upon the gauge is contained in the variables of gauge invariant functional $\hat{\Gamma}_R(A') = \tilde{\Gamma}_R(\{\vartheta\}, A)$. The explicit form of representation (25) enables one to draw a conclusion that the generating functional $\tilde{\Gamma}_R$ on its extremals

$$\frac{\delta\tilde{\Gamma}_R}{\delta A^\imath} = 0 \tag{26}$$

does not depend upon the gauge. Indeed, variation of $\tilde{\Gamma}_R$ is written by the relation for variation of gauge condition with respect to the

one of parameters ϑ:

$$\delta_\vartheta \tilde{\Gamma}_R = \frac{\delta \hat{\Gamma}_R}{\delta A'^\imath} \frac{\partial A'^\imath}{\partial \vartheta} \delta \vartheta \tag{27}$$

In view of nondegeneracy of the parametrization $A' = A'(\{\vartheta\}, A)$ one obtains simultaneously with (26) $\frac{\delta \hat{\Gamma}_R}{\delta A'^\imath} = 0$. Consequently $\tilde{\Gamma}_R$ depends upon $\{\vartheta\}$ off shell only.

As regards the example of Einstein gravity considered above the general relations of this section assume the following form. The condensed notation of indices are described by (4), the fields $A^\imath$ corresponding to the metric tensor $\bar{g}_{\mu\nu}(x)$. Meanwhile the gauge algebra structural coeffitientse $F^\gamma_{\alpha\beta}$ in (21) are assotiated with the functions $f^\lambda_{\rho\sigma}(x, y, z)$ in (5) and $\varepsilon_\imath = \varepsilon_\alpha = 0$.

Concluding, note that the resultant form for the divergent part of one-loop effective action (20) does not coincide with the result of paper [17], where the following term is present:

$$\zeta \cdot k \cdot R_{\mu\nu\lambda\sigma} R^{\lambda\sigma\delta\alpha} R^{\mu\nu}{}_{\delta\alpha}. \tag{28}$$

The reason for this structure to appear is wrong usage of the relation (15). In [17] the contributions of ghost and gauges fields to $\Gamma_{1,div}$ have not been calculated correctly, and therefore the term (28) remains in $\Gamma_1(\zeta)$.

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